Hall Ticket Number:

Code No. : 11214 S O

VASAVI COLLEGE OF ENGINEERING *(AUTONOMOUS)*, HYDERABAD

Accredited by NAAC with A++ Grade

B.E. I-Semester Supplementary Examinations, August-2023

Calculus and Linear Algebra

(Common to CSE, AIML & IT)

Time: **3 hours**

Max. Marks: **60**

*Note: Answerallquestions from**Part-A** and any **FIVE** from **Part-B***

Part-A (10× 2 = 20 Marks)

Q. No.	Stem of the question	M	L	CO	PO
1.	Define curvature and centre of curvature	2	1	1	1,2,12
2.	Find the radius of curvature at (0,0) for the curve $x^3 + y^3 - 2x^2 + 6y = 0$	2	1	1	1,2,12
3.	Write the necessary condition for $f(x, y)$ to be maximum and minimum.	2	1	2	1,2,12
4.	Expand e^{xy} at $(1,1)$ up to the second degree terms.	2	1	2	1,2,12
5.	Define Basis of a vector space.	2	1	3	1,2,12
6.	State Rank-Nullity theorem.	2	1	3	1,2,12
7.	Define Rank of a matrix.	2	1	4	1,2,12
8.	Find the eigen values of $\begin{bmatrix} 2 & 1 & -3 \\ 0 & -4 & 5 \\ 0 & 0 & 3 \end{bmatrix}$.	2	2	4	1,2,12
9.	Define Absolute and Conditional convergence.	2	1	5	1,2,12
10.	Test for the convergence of the series $\sum \left(\frac{n+1}{n}\right)^{n^2}$.	2	2	5	1,2,12

Part-B (5×8 = 40 Marks)

Q. No.	Stem of the question	M	L	CO	PO
11. a)	Show that the equation of evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$.	4	2	1	1,2,12
b)	Find the centre of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.	4	2	1	1,2,12
12. a)	If $u = f(y - z, z - x, x - y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	4	2	2	1,2,12
b)	Find the maximum value of $x^2 + y^2 + z^2$ subject to $x + y + z = a$.	4	3	2	1,2,12

Contd... 2

		M	L	CO	PO
13. a)	Define the linear transformation $T : R^4 \to R^3\ by$ $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \left(\begin{bmatrix} a+b \\ b-c \\ a+d \end{bmatrix}\right)$. Find a basis for the null space of T and its dimension.	4	3	3	1,2,12
b)	Let $V = \left\{\begin{bmatrix} t \\ 1+t \end{bmatrix} / t \in R\right\}$ Define $\begin{bmatrix} t_1 \\ 1+t_1 \end{bmatrix} \oplus \begin{bmatrix} t_2 \\ 1+t_2 \end{bmatrix} = \begin{bmatrix} t_1+t_2 \\ 1+t_1+t_2 \end{bmatrix}$ and $c \odot \begin{bmatrix} t \\ 1+t \end{bmatrix} = \begin{bmatrix} ct \\ 1+ct \end{bmatrix}$. Find the additive identity and inverse.	4	3	3	1,2,12
14. a)	Diagonalize the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ using similarity transformation.	4	3	4	1,2,12
b)	If $B = \left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$ be an ordered basis then find an Orthonormal basis for R^3 by using Gram-Schmidt process, where the inner product is the dot product.	4	3	4	1,2,12
15. a)	Discuss the convergence of the series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \cdots$	4	3	5	1,2,12
b)	Show that the series $\sum(-1)^n \frac{1}{5\sqrt{n}}$ is conditionally convergent.	4	2	5	1,2,12
16. a)	Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.	4	3	1	1,2,12
b)	If $z = f(x,y)$, where $x = e^u + e^{-v}, y = e^{-u} - e^v$. Show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.	4	2	2	1,2,12
17.	Answer any ***two*** of the following:				
a)	Define a mapping $T: P_3 \to P_2$ by $T(p(x)) = p'(x)$ where $p'(x)$ is the derivative of $p(x)$. Show that T is a Linear Transformation.	4	3	3	1,2,12
b)	Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.	4	3	4	1,2,12
c)	Test the convergence of the series $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \cdots$	4	3	5	1,2,12

M : Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

i)	Blooms Taxonomy Level – 1	20%
ii)	Blooms Taxonomy Level – 2	30%
iii)	Blooms Taxonomy Level – 3 & 4	50%

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